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Reggeized $\pi\rho$ Mass Enhancement in the A_1 Region*

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Mass distributions for the $\pi\rho$ final state in the reaction $\pi N\to\pi\rho N$ are calculated from a Regge-pole-exchange model. Compared with the results of the Drell-Deck-type models, significantly increased $\pi\rho$ mass peaking in the A_1 region is predicted; calculated widths are consistent with results of recent experiments on the A_1.

ELEMENTARY one-pion-exchange diagrams of the Deck-Drell-Hiida type[1] have been studied recently for the purpose of calculating background distributions for the reaction $\pi N\to\pi\rho N$.[2] In this paper, the results of a Regge-pole-exchange model calculation of the mass- and momentum-transfer distributions are presented; the method yields, in comparison with elementary exchange models, more pronounced enhancement of final $\pi\rho$ invariant masses in the A_1 (mass$\approx$1.08 BeV/c^2) region. Computed enhancement widths are consistent with the results of recent experiments on the A_1.

The basic assumption here is that $\pi N\to\pi\rho N$ proceeds primarily via doubly peripheral collisions of the type diagrammed in Fig. 1, where I and II are Regge-pole exchanges. We define, in terms of Fig. 1, where the p's and q's are four-momenta, five independent invariant variables upon which the amplitude depends:

$$s=(p_1+p_2)^2,\quad s_1=(q+q_1)^2,\quad s_2=W^2=(q+q_2)^2,$$
$$t_1=(q_1-p_1)^2,\quad t_2=(q_2-p_2)^2.$$

In their work on multiple-production theory, Bali, Chew, and Pignotti[3] observed that for such diagrams, denoting the respective Regge trajectories by $\alpha_{\rm I}(t_1)$ and $\alpha_{\rm II}(t_2)$, one has

$$d\sigma\propto(s_1/s_2)^{\alpha_{\rm I}-\alpha_{\rm II}}d\ln(s_1/s_2)\tag{1}$$

for s_1, s_2, and s all large. In particular, suppression of large s_2 is greatest when $\alpha_{\rm I}$ and $\alpha_{\rm II}$ are the highest and lowest lying, respectively, consistent with quantum-number demands of the diagram. Thus, for the mass labeling given in Fig. 1(a), if $\alpha_{\rm II}$ is the pion trajectory and $\alpha_{\rm I}$ the Pomeranchuk trajectory, large values of the $\pi\rho$ subenergy will be strongly suppressed, whereas if $\alpha_{\rm I}$ is the P' or ρ trajectory, similar but less marked large s_2 damping will result. Moderate damping of large s_2 will also occur for Fig. 1(b), where, for example, $\alpha_{\rm II}=\alpha_\rho$, and $\alpha_{\rm I}$ is the Pomeranchuk trajectory.

The cross section associated with $\pi N\to\pi\rho N$ is written

$$d\sigma=(1/2\pi)^5(1/4F_I)|M|^2d\phi_3,\tag{2}$$

where F_I is the invariant flux, equal to the product of the target nucleon mass, m_N, and the incident pion momentum (lab), and $d\phi_3$ denotes the phase space.[4]

The Regge-pole hypothesis is adopted for the absolute square of the invariant amplitude M summed over final spins and averaged over initial spins.[3] Therefore, the contribution from Fig. 1(a), in a form which displays only the pion Reggeization explicitly ($\alpha_{\rm II}=\alpha_\pi$), is

$$|M|^2=|f_\pi(t_2)S_\pi(t_2)M_{\pi N}'|^2(\cosh\xi_2)^{2\alpha_\pi(t_2)},\tag{3}$$

where the Reggeized pion propagator[5] is

$$S_\pi(t_2)=\frac{\pi\alpha_\pi'}{\sin\pi\alpha_\pi}\left(\frac{1+e^{-i\pi\alpha_\pi}}{2}\right)\frac{(2\alpha_\pi+1)\Gamma(\alpha_\pi+\frac12)}{(\sqrt{\pi})\Gamma(\alpha_\pi+1)},\tag{4}$$

$$\alpha_\pi'=d\alpha_\pi/dt_2|_{t_2=m_\pi^2},\tag{5}$$

$$\cosh\xi_2=-2t_2\lambda_2^{-1/2}\lambda_3^{-1/2}[s_2-t_1-m_\pi^2-\tfrac12t_2^{-1}(m_\rho^2-m_\pi^2-t_2)(t_1+t_2-m_\pi^2)],\tag{6}$$

$$\lambda_2=t_1^2+t_2^2+m_\pi^4-2t_1t_2-2m_\pi^2(t_1+t_2),\tag{7}$$

$$\lambda_3=m_\rho^4+m_\pi^4+t_2^2-2m_\rho^2m_\pi^2-2t_2(m_\rho^2+m_\pi^2).\tag{8}$$

Present experimental evidence is consistent with a small slope, if any, for the Pomeranchuk trajectory.[6]

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[1] S. D. Drell and K. Hiida, Phys. Rev. Letters **7**, 199 (1961); R. T. Deck, *ibid.* **13**, 169 (1964); U. Maor and T. A. O'Halloran, Phys. Letters **15**, 281 (1965); U. Maor, Ann. Phys. (N. Y.) **41**, 456 (1967); L. Stodolsky, Phys. Rev. Letters **18**, 973 (1967); M. Ross and Y. Yam, *ibid.* **19**, 546 (1967).

[2] At incident π^+ lab momentum 3.65 BeV/c: Goldhaber *et al.*, Phys. Rev. Letters **12**, 336 (1964); B. C. Shen *et al.*, *ibid.* **15**, 731 (1965). 1.5- to 4.2-BeV/c π^-: S. U. Chung *et al.*, Phys. Rev. Letters **12**, 621 (1964); S. U. Chung, thesis, University of California Radiation Laboratory Report No. UCRL-16981, 1966 (unpublished). 8.0 BeV/c π^+: Aachen-Berlin-CERN collaboration, Phys. Letters **19**, 608 (1965); **22**, 112 (1966). 8.0 BeV/c π^-: Cason *et al.*, Phys. Rev. Letters **18**, 880 (1967). 11.0 BeV/c π^-: Genoa-Hamburg-Milan-Saclay collaboration, Nuovo Cimento **47**, 675 (1967); 1967 report (unpublished).

[3] N. F. Bali, G. F. Chew, and A. Pignotti, Phys. Rev. Letter **19**, 614 (1967); Phys. Rev. **163**, 1572 (1967). The multi-Regge-pole exchange hypothesis has also been studied by K. A. Ter-Martirosyan, Nucl. Phys. **68**, 591 (1965); H. M. Chan, K. Kajantie, and G. Ranft, Nuovo Cimento **49**, 157 (1967); F. Zachariasen and G. Zweig, Phys. Rev. **160**, 1322 (1967); **160**, 1326 (1967); and others.

[4] Use of Toller variables, as in Ref. 3, is not essential for writing the phase space, but is important for the Reggeization procedure.

[5] See, for example, E. J. Squires, *Complex Angular Momentum and Particle Physics* (W. A. Benjamin, Inc., New York, 1963).

[6] For discussion and references, see G. F. Chew, Comments on Nuclear and Particle Physics **1**, 121 (1967).

Moreover at energies now accessible, exchanges other than the Pomeranchuk in leg I of Fig. 1(a) are expected to contribute. Consequently, a Reggeized form for $M_{\pi N}'$ is not adopted[7]; rather, the off-mass-shell πN-scattering amplitude is approximated by the on-shell amplitude, which in turn is related to the πN differential cross section characterized at high energy by a pronounced diffraction peak at small four-momentum transfer t_1. Therefore let

$$|M_{\pi N}'|^2=(d\sigma/d\Omega)_0 e^{At_1}, \tag{9}$$

where A is the slope, on a logarithm plot, of the πN elastic differential cross section. On the basis of the optical theorem one writes

$$(d\sigma/d\Omega)_0=\lambda_0\sigma_{\pi N}^2; \tag{10}$$

$$\lambda_0=[s_1-(m_N-m_\pi)^2][s_1-(m_N+m_\pi)^2], \tag{11}$$

and $\sigma_{\pi N}$ is the πN total cross section. This procedure is similar to that of others,[8] and is in agreement with the experimental observations in $\pi N\to\pi\rho N$.[2]

Over-all normalization is achieved by requiring that in the limit $t_2\to m_\pi^2$, Eq. (3) reduce to that appropriate to the exchange of an elementary pion, viz.,

$$|M_{\rm elem}|^2=g^2\frac{(m_\rho^2-4m_\pi^2)}{(t_2^2-m_\pi^2)^2}|M_{\pi N}'|^2. \tag{12}$$

Here g is the effective $\pi\pi\rho$-coupling constant; $(g^2/4\pi)=2.2$. Because $\alpha_\pi(t_2)\to 0$ as $t_2\to m_\pi^2$, consistency of Eqs. (3) and (12) requires

$$f_\pi(m_\pi^2)=g^2(m_\rho^2-4m_\pi^2). \tag{13}$$

A curved pion trajectory of the Pignotti type[9] was used. However, a linear trajectory yields similar results:

$$\alpha_\pi=-(m_\pi^2-t_2)[m_\pi^2-t_2+1]^{-1}. \tag{14}$$

After the threshold factors are removed from $f_\pi(t_2)$ and certain factors extracted from the Γ functions in Eq. (4), the final form obtained for the contribution of Fig. 1(a) is

$$|M_a|^2=g^2(m_\rho^2-4m_\pi^2)\lambda_0[\pi\sigma_{\pi N}(s_1)]^2(1+\alpha)^2e^{At_1}$$
$$\times\frac{\beta(t_2)}{2(1-\cos\pi\alpha)}[s_0^{-1}\{s_2-t_1-m_\pi^2$$
$$-\tfrac{1}{2}t_2^{-1}(m_\rho^2-m_\pi^2-t_2)(t_1+t_2-m_\pi^2)\}]^{2\alpha_\pi}. \tag{15}$$

Here $\beta(t_2)$ is a smooth function equal to unity at $t_2=m_\pi^2$. In usual Regge-pole fits[10] it is taken as a decreasing function as $(-t_2)$ increases. The choice of s_0 is somewhat arbitrary; usually $s_0\approx 1$ BeV^2.

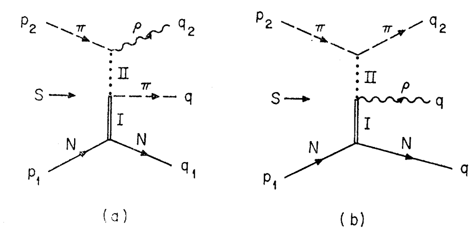

FIG. 1. Regge-pole-exchange diagrams which give rise to enhancement of low $\pi\rho$ masses.

Because results of calculations with the Deck-type matrix element [Eq. (12)] are fairly well known, results obtained by the two methods are compared in Fig. 2 for incident pion lab momenta 8.0 and 11.0 BeV/c. For simplicity, $\sigma_{\pi N}$ at both momenta for both Regge [Eq. (15)] and non-Regge [Eq. (12)] matrix elements was fixed at 29 mb. For both matrix elements at 8 BeV/c, $A=8.0$ (BeV)$^{-2}$, and at 11 BeV/c, $A=9.0$ (BeV)$^{-2}$. In all computations $s_1>1.8$ (BeV)^2 in order to exclude the (3,3) isobar region. Otherwise all integrations were performed over the entire regions allowed kinematically. In the spirit of most Deck-type calculations in which no auxiliary damping factors are introduced at the $\pi\pi\rho$ vertex, s_0 and $\beta(t_2)$ were both set equal to unity.[11]

The total $\pi\rho$ production cross sections obtained by the different methods were the same, 0.15 mb, at both momenta. Using the Reggeized matrix element, one obtains distributions peaked slightly lower (1.08 versus 1.15 BeV/c^2) with full widths at half-maximum of 450 MeV versus 700 to 800 MeV in the Deck approach. Differential cross sections, $d\sigma/dt_1$ in "the A_1 region, $0.96<W<1.2$," were also computed. In this respect there is little difference between the two models; plots of $\log(d\sigma/dt_1)$ versus t_1 yield straight lines in both cases. The slopes at 8.0 BeV are 10.0 (BeV)$^{-2}$ and at 11.0 BeV/c are 11.0 (BeV)$^{-2}$, in agreement with experiment.[2,12]

Both the elementary pion-exchange and Reggeized exchange models, as discussed here, yield total cross sections approximately $\frac{1}{2}$ those measured in the laboratory. The agreement may be improved as follows: (a) by including the energy dependence of $\sigma_{\pi N}$ and of A in Eq. (15) [this is equivalent to adding the contribution of exchanges other than the Pomeranchuk in leg I of Fig. 1(a)]; (b) by including the effects of diagrams with ρ exchange, as in Fig. 1(b). Each of

[7] This point is discussed further in footnote 12; use of the non-Regge form also facilitates normalization, as will be seen.

[8] See papers by Deck, Maor, Stodolsky, and Ross and Yam in Ref. 1.

[9] A. Pignotti, Phys. Rev. Letters **10**, 416 (1963); R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965).

[10] C. B. Chiu, R. J. N. Phillips, and W. Rarita, Phys. Rev. **153**, 1485 (1967), and references therein.

[11] Comments on this choice are made later in this paper. In a detailed comparison with data, s_0 and $\beta(t_2)$ could be fixed by fitting the experimental distribution in t_2.

[12] Calculations employing Reggeization of both the pion and Pomeranchuk exchanges in Fig. 1(a) (with $\alpha_P\equiv 1.0$) have also been performed: the exponential damping factor at the N-N-Pomeranchuk vertex was chosen as in the previous paragraph. Final $\pi\rho$ mass distributions were peaked at the same position and had the same widths as those of the semi-Reggeized model just discussed.

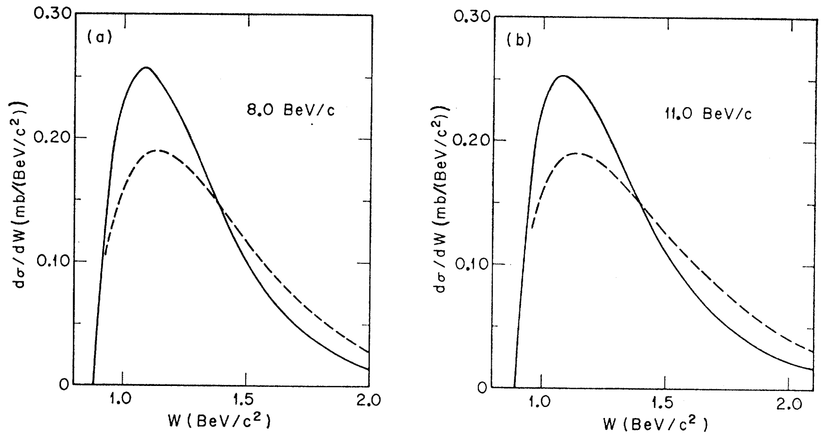

FIG. 2. Comparison of $\pi\rho$ mass distributions calculated from pion Regge-pole-exchange model (solid line) and elementary pion-exchange Deck-type model (dashed line) for the reaction $\pi N \rightarrow \pi\rho N$ at incident pion momenta: (a) 8.0 BeV/c and (b) 11.0 BeV/c. W is the invariant mass of the $\pi\nu$ system.

these contributes about 0.1 mb to the total cross section in the form of an enhancement, 800 to 900 MeV/c^2 wide, peaked near $W=1.2$ BeV/c^2. After inclusion of both, the full width increased to 500 MeV/c and the peak location shifted to $W=1.1$ BeV/c^2. Widths of this size are consistent with those of recent experimental distributions obtained at these incident momenta.[2,13] More detailed analyses keyed to the characteristics of given experiments would be very valuable to determine what fraction of reported A_1 peaks can actually be accounted for by this Regge-pole-exchange model.

Minor ambiguities deserve comment. By taking $s_0<1$ (BeV)^2 or $\alpha_\pi'>1$ (the value used here), or by introducing the form factor $\beta(t_2)$, narrower widths can be obtained. However, the requirement that the location of the experimental A_1 enhancement be reproduced limits freedom; it is unlikely that a width less than 350 MeV/c^2 could be realized. The over-all energy dependence of the enhancement parameters was studied: at 30.0 BeV/c with $A=10.0$ and $\alpha_\pi'=1.0$, $d\sigma/dW$ peaks at $W=1.1$ BeV/c^2 and has a full width of 550 MeV/c^2.

The possibility that the results reported here might also be obtained via the traditional momentum-transfer–dependent form-factor modification[14] of the elementary one-pion-exchange (OPE) model was investigated. A form factor of the type

$$F(t_2)=P(t_2)e^{Bt_2}, \qquad (16)$$

in which $P(t_2)$ is a polynomial, was introduced as a multiplicative factor on the right-hand side of Eq. (12). Normalization was fixed, again, by requiring that

$$F(m_\pi^2)=1.0, \qquad (17)$$

and then $P(t_2)$ and the constant B were adjusted so that the modified elementary OPE matrix element gave the same t_2 distribution $d\sigma/dt_2$ as the (unmodified) Regge-type matrix element [Eq. (15)]. The resulting mass distribution $d\sigma/dW$ was then computed and seen to bear a relationship to that of the Regge-type model similar to those shown in Fig. 2; thus the elementary model will not yield the same results.

An analytical understanding of the increased low-mass enhancement obtained in the Regge model can be obtained by comparing Eqs. (12) and (15) at various values of s_2. For s_2 small, the right-hand side of Eq. (15) decreases less rapidly with increasing $(-t_2)$ than does the right-hand side of Eq. (12) and thus yields a greater cross section; whereas as s_2 gets large, the right-hand side of Eq. (15) is dominated by its last factor which, for small values of the momentum transfers, is essentially $(s_2/s_0)^{2\alpha_\pi}$, thus suppressing the larger s_2 because α_π is always negative. As a check on the applicability of the Regge model, the doubly differential distribution $d\sigma/ds_2dt_2$ should be examined experimentally.

The effect described here is relevant also to the computation of threshold enhancements in the mass of certain particle pairs in other three-body final-state processes such as $Kp\rightarrow \pi pK^*(890)$, $pp\rightarrow pp\pi$, $\pi p\rightarrow \pi\pi p$, and $\pi p\rightarrow \pi p\Delta$. A detailed fit to data on the reaction $pp\rightarrow \pi p\Delta$ is in progress.

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It is a pleasure to thank Professor Geoffrey Chew, Dr. N. Bali, and Dr. A. Pignotti for valuable discussions.

[13] Preliminary results [Phys. Letters **22**, 112 (1966)] of the Aachen-Berlin-CERN collaboration gave evidence of a much narrower A_1; the valley reported there between A_1 and A_2 peaks has since vanished with better statistics (private communication from D. R. O. Morrison).

[14] See, for example, E. Ferrari and F. Selleri, Phys. Rev. Letters **7**, 387 (1961); Nuovo Cimento, Suppl. **24**, 453 (1962).